Probabilistic Modeling of Dynamic U-Factor

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ABSTRACT

A probabilistic model of thermal transmittance (U-factor) for a building with lightweight construction is presented. Steady-state conditions of airflow and heat flow through the structure are assumed. Three cases, dependent on the direction of the airflow through the envelope, are considered: tight envelope, infiltration, and exfiltration. Thermal transmittance is treated in the model as a random variable varying with the changes of microclimatic conditions at the site of a building.

As an example, the test house situated in two different climatic zones is considered. The probability density functions of U-factor are approximated on the basis of probability density functions of climatic parameters and the air change rate. First-order reliability method (FORM) techniques are used as a tool for the probabilistic approximation.

INTRODUCTION

For some building technologies, especially those involving lightweight timber frame with mineral wool filling and loose mineral wool layers for roof insulation, one can speak about the dependence of the thermal properties of building components on air infiltration. It is very clear for the so-called “dynamic wall” (Anderlind and Johansson 1983), specially designed to save energy. The interaction between thermal transmittance and airflow through the components should be considered while calculating heat loss through a building envelope. The thermal transmittance becomes the most interesting parameter that can vary with the climatic parameters. A modeling approach based on probabilistic methods is proposed (Pietrzyk et al. 2003a, 2004), which seems very well suited for handling the natural driving forces governing the rate of airflow through a building envelope and the value of thermal transmittance.

The probabilistic model of thermal transmittance is shown in Figure 1. It includes a limit state approach based on first-order reliability method (FORM) techniques, which gives opportunity to directly calculate the probability of random variable (U-factor) exceeding the assumed serviceability limit, as well as to approximate the probability density function of that variable (Pietrzyk 2000). The load random variables are selected from the many parameters influencing thermal transmittance. It means that only some of the parameters and mechanisms influencing dynamic performance of a building envelope have been taken into account in the model.

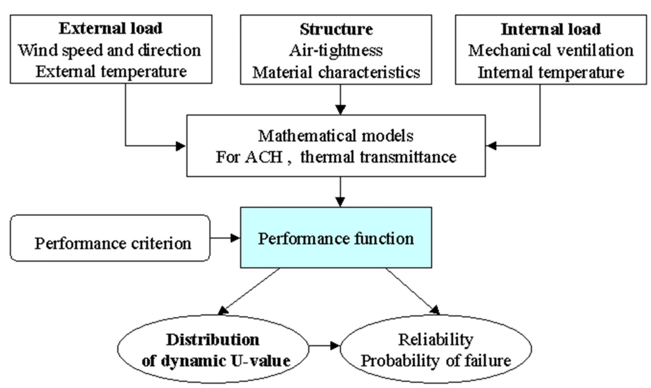

Figure 1 *Probabilistic modeling of thermal transmittance.*

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However, the aim for the authors in this paper is not to deliver a complete product for the practitioners but to demonstrate a new modeling approach in analyzing dynamic performance of building envelopes.

FORMs provide effective estimation of the probability of failure, i.e., the probability that a certain design requirement is not fulfilled, without the need—but leaving the possibility—to evaluate the probability density function of the output variable. Applying FORM offers savings in computer processing time as compared to, for example, Monte Carlo simulations. Fundamentals of reliability analysis together with the description of FORM can be found in Ditlevsen and Madsen (1996), Haldar and Mahadevan (2000), Nowak and Collins (2000), and Czmoch (1998).

THERMAL TRANSMITTANCE—A VIEW OF DETERMINISTIC APPROACH

The averaged thermal transmittance of a building component U_i^0 is given by the following equation (Hagentoft 2001):

$$U_i^0 = \frac{1}{R_i} = \frac{1}{\frac{1}{\alpha_{ex}} + \frac{1}{\alpha_{in}} + \sum_{j=1}^{m} \frac{d_j}{\lambda_{c,j}}} \quad (1)$$

where

R_i = total thermal resistance of the ith element of the envelope (m^2K/W)

α_{ex} = external heat transfer coefficient (W/m^2K)

α_{in} = internal heat transfer coefficient (W/m^2K)

d_j = thickness of the jth layer of the ith element of the envelope (m)

$\lambda_{c,j}$ = thermal conductivity estimated for certain humidity and temperature of the material of the jth layer of the ith element of the envelope (W/mK)

Thermal conductivity increases with the actual temperature of the material. The results of the measurements for the insulation materials presented by Wahlgren (2001) and Serkitjis (1995) show this phenomenon.

The Nusselt number Nu describes the effect of convective flows (leakage and interstitial convection) on the thermal performance of a structure. The Nusselt number for the ith element is defined as in Kohonen et al. (1985).

$$\mathrm{Nu}_i = \frac{\Phi_{t,i}}{\Phi_{t0,i}} \quad (2)$$

where

$\Phi_{t,i}$ = transmission heat flow through the ith element (with leakages) (kW)

$\Phi_{t0,i}$ = transmission heat flow through the ith element estimated for the impermeable surface (without leakages) (kW)

One has to point out that the so-called transmission heat flow includes the effect of infiltration on the heat conduction.

The Nusselt number is equal to one for the element without convection flow. The value of the Nusselt number depends on the velocity of the airflow through the insulation, the direction of the flow, and the thickness and density of the insulation. It can be described by the expression below (Hagentoft 2001), where the cases of infiltration and exfiltration are given by different equations.

$$\mathrm{Nu} = \xi \frac{e^{\xi}}{e^{\xi} - 1} \quad (3)$$

and, for the case of exfiltration,

$$\xi = \frac{c_a \rho_a Q}{U^0 A}, \quad (4)$$

and, for the case of infiltration,

$$\xi = -\frac{c_a \rho_a Q}{U^0 A}, \quad (5)$$

where

ρ_a = air density (kg/m^3),

c_a = specific heat capacity of air (at a constant pressure) (J/kgK),

U^0 = thermal transmittance (see Equation 1) (W/m^2K),

A = area (m^2),

Q = airflow rate (m^3/s).

Infiltration Case

In the case of infiltration, the cold air from the outside flows through the cracks and openings in the envelope into the building, while the heat flows in the opposite direction, as shown in Figure 2. The transmission heat loss is lower, compared to pure conduction, because the conduction heat is partially used to warm up the infiltrating air on the leakage routes from the outer surface of the structure to the inner one. The relative reduction of transmission heat flow at the outer surface is described by the Nusselt number (Nu < 1) (Kohonen et al. 1985). The inflow temperature is equal to the temperature outside of the building. The temperature of the internal surface of the wall is lower than the temperature of the internal surface of a tight envelope, as shown in Figure 2.

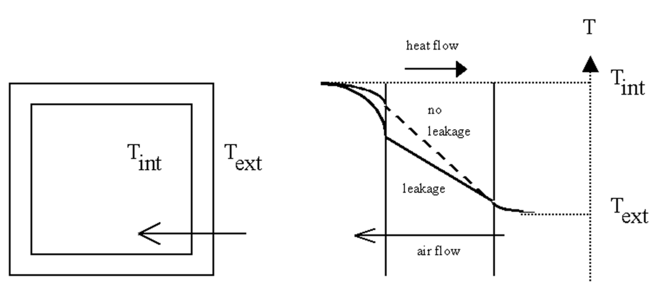


Figure 2 *Illustration of heat flow through the leaky wall—infiltration.*

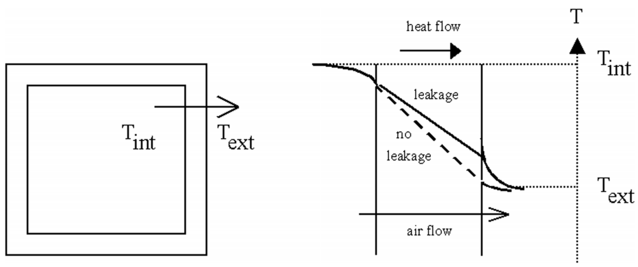


Figure 3 *Illustration of heat flow through the leaky wall—exfiltration.*

Exfiltration Case

In the case of exfiltration, the hot air from the inside flows through the cracks and the openings in the envelope out of the building, and the heat also flows in the same direction as the exfiltrating air. Exfiltration increases transmission heat flow at the outer surface of the building envelope as compared to the case of pure conduction. It means that the Nusselt number becomes greater than one (Nu > 1). The inlet temperature is equal to the temperature inside the building, while the outlet temperature is higher than in the case of a tight envelope, as shown in Figure 3.

The overall average thermal transmittance of a building envelope, including the influence of flow passing through the insulation on conductive heat losses, called dynamic U-factor, is defined as

$$U = \frac{1}{A_{tot}} \sum_{i=1}^{n} \mathrm{Nu}_i U_i^0 A_i \, , \tag{6}$$

where

A_{tot} = area of a building envelope (m^2),

Nu_i = Nusselt number defined by Equation 2,

U_i^0 = average thermal transmittance of the *i*th element of the building envelope (W/m^2K),

U = dynamic U-factor (W/m^2K).

THERMAL TRANSMITTANCE—A VIEW OF PROBABILISTIC APPROACH

Uncertainties Related to Thermal Transmittance

There are many sources of uncertainties related to thermal performance of the building envelope.

- People: bad workmanship, improper or too complicated design solutions
- Material: deficiencies in the material, variations of material characteristics in space or time
- Actual conditions of using the house: external and internal environmental load

The usual way to deal with them is deterministic evaluation of common deviations from the ideal situation when the nominal U-factor of the design of the building envelope is considered in full-scale, resulting in the form of practical thermal transmittance U_p. Those deviations are estimated in the form of corrections to the nominal U-factor considering the influence of thermal bridges ΔUf, bad workmanship ΔUg, complicated insulation ΔU_k, and the influence of rain and wind for special construction of the roof ΔU_w, as specified in the Swedish code of practice (Boverkets Byggregler 1994).

$$U_p = \frac{1}{R_p} + \Delta U_f + \Delta U_g + \Delta U_k + \Delta U_w \tag{7}$$

Probabilistic Modeling of U-Factor

Probabilistic model of thermal transmittance presented in this paper takes into account only some of the uncertainties related to the environmental impact (external load) and the way of using the house (internal load), as shown in Figure 1.

The following assumptions have been adopted:

- low-rise building
- lightweight construction
- single ventilation zone
- exchanging of air is carried out by filtration through the envelope
- single temperature zone
- steady-state conditions of airflow and heat flow
- leakage area is treated as uniformly distributed over each building component

Generally, the model can be also applied to the cases with nonhomogeneous material characteristics (for example, density or thickness of insulation) or can be expanded to include, for example, influence of wind or radiation on external heat transfer coefficient. Variability of the U-factor should be then approximated, accounting for the distributions of those characteristics.

U-Factor for a Tight Envelope—U^0. Thermal transmittance (U-factor) is a measure of the rate of heat transfer through a construction under standard conditions. It means that it is calculated by making certain assumptions concerning temperature and humidity of the material, rates of heat transfer at surfaces, etc. The standard assumptions are intended to represent practical conditions as far as possible. Nevertheless, these parameters are assumed to be constant while, in reality, they are varying in time. To illustrate how the variation of the above mentioned parameters can be incorporated in the probabilistic model, the case of temperature dependence is described below.

Thermal transmittance of a building component is estimated at a certain temperature (in Sweden, the temperature is taken as 10°C) of the material. The actual temperature of the insulating material can vary with the changes in the climatic conditions. The dependence of the U-factor on the temperature of the material, T_w, can be approximated by the following expression (Bankvall 1972):

$$U^0 = U^0_{10} + x(T_w - 10) \tag{8}$$

where

U^0_{10} = U-factor for the material in temperature of 283K (W/m^2K)

x = parameter depending on the density of the insulation (W/m^2K^2)

An example of relationship between thermal conductivity of a porous material λ_c and its temperature T_w for a fibrous insulation with a density of 16.4 kg/m^3 is shown in Figure 4 (Bankvall 1972).

For the constant internal temperature T_{int} = 20°C, the mean temperature of insulation can be approximated by

$$T_w = (20 - \Delta T/2)\ , \tag{9}$$

where ΔT is the difference between internal and external temperature.

Assuming an internal temperature of 20°C and that the external temperature varies between –20°C and 13°C during the heating season, the mean temperature of the insulation is in the range of 0°C to 17°C. Surface heat transfer coefficients are also temperature dependent. The thermal transmittance is a function of the thermal conductivity, and the internal and external heat transfer coefficients can vary about the design value by 5% for low-density insulation. Finally, accounting for Equations 8 and 9, thermal transmittance for a tight envelope U^0 can be given by

$$U^0 = U^0_{10} + x\left(10 - \frac{\Delta T}{2}\right)\ . \tag{10}$$

For a normally distributed temperature difference (Pietrzyk 2000) the mean and the standard deviation of U-factor are estimated as

$$\begin{aligned} \mu_{U^0} &= U^0_{10} + x\left(10 - \frac{\mu_{\Delta T}}{2}\right) \\ \sigma_{U^0} &= \frac{x}{2}\sigma_{\Delta T}\ . \end{aligned} \tag{11}$$

The thermal transmittance U^0 is treated in the model as a normally distributed random variable with the mean value and the standard deviation given by Equation 11, expressing variations of the mean U-factor of n elements of the building envelope U^0_{10} caused by the changing outdoor temperature. The mean (area weighted) thermal transmittance is given by the following equation:

$$U^0_{10} = \frac{1}{A_{tot}} \sum_{i=1}^{n} U^0_{10,i} A_i \tag{12}$$

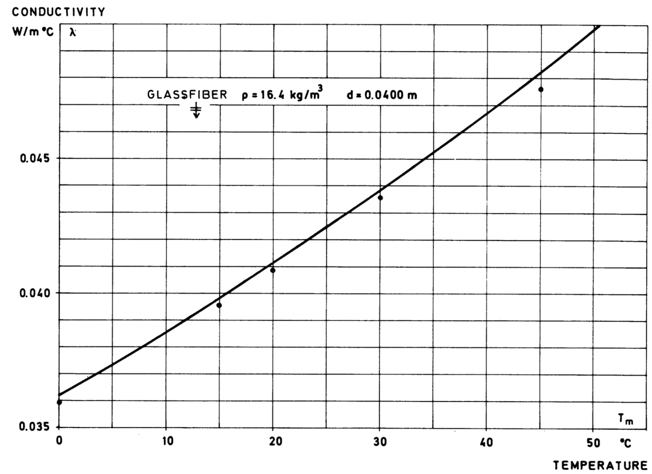


Figure 4 *Thermal conductivity of fibrous insulation as a function of mean temperature of the material (courtesy of Bankvall [1972]).*

The mean thermal transmittance of the ith component of a building envelope $U^0_{10,i}$ (see Equation 12) could even include the deterministic corrections presented in Equation 7.

U-Factor for a Leaky Envelope (Dynamic U-Factor)—*U.* It has been assumed for a tight building that variations of U^0 are caused mainly by the changes of external temperature. For a leaky building, airflow through the building envelope (governed by random climatic conditions) can also change the actual U-factor (Kohonen et al. 1985), depending on the length of the leakage route and the thermal properties of the wall around the crack. The influence of the convective flow on conductive heat losses is described by the Nusselt number (see Equation 2). Variations of the actual U-factor are caused by a climate-dependent velocity of the flow through insulation (see Equations 3 to 6). The actual value of overall average thermal transmittance depends on the temperature in the insulation as well as on the Nusselt number, which can be expressed by the function of the airflow and the thermal transmittance U^0 (see Equations 3 to 5). Eventually, thermal transmittance U^0 and air change rate have been chosen to be the input random variable for the model of a leaky building envelope.

Dynamic thermal transmittance depends also on the direction of airflow through a structure and, therefore, is specified separately for infiltration and exfiltration case.

The decision of which case applies for the chosen building component is governed by

- wind velocity and wind direction,
- location of the component with respect to the neutral pressure layer, and
- mechanical ventilation (for example, exhaust ventilation system, using the openings in the building envelope as inlet of infiltrating air).

Approximation of a Probability Density Function of Thermal Transmittance

Approximation of the probability density functions of a dynamic U-factor is carried out with the help of FORM techniques. The probability density function of dynamic U-factor is evaluated using FORM sensitivity analysis (Pietrzyk 2000; Pietrzyk and Hagentoft 2003a). It depends on statistical parameters of the joint distribution of two random variables: thermal transmittance U^0 (see Equation 1) and air change rate (ACH). Stochastic information is limited to the parameters of marginal probability density functions of those variables and the correlation coefficient between them.

The air exchange carried out by filtration through the envelope is evaluated with the help of probabilistic model PROMO described by Pietrzyk (2000) and in Pietrzyk and Hagentoft (2003b). It takes into account the influence of natural forces (wind, temperature) and mechanical forces. For a naturally ventilated building, the model used can be described by the following equation:

$$ACH_d = \sqrt{s_1 \Delta T^2 + s_2 |\Delta T| + s_3 |\Delta T|^{1.5} + w_{d,1} v_d^4 + w_{d,2} v_d^2 + w_{d,3} v_d^3} \tag{13}$$

where

$\Delta T = T_{int} - T_{ext}$	=	temperature difference across the envelope
v	=	wind speed
d	=	wind direction
s_1, s_2, s_3	=	deterministic coefficients dependent on the parameters of the flow balance for the stack effect
$w_{d,1}, w_{d,2}, w_{d,3}$	=	deterministic coefficients dependent on the flow balance for various wind directions d

The probabilistic distributions of the air change rate are approximated on the basis of climatic parameter distributions typical for the site: wind speed and direction, internal-external temperature difference, and data concerning the building, its environment, and its using like airtightness of the envelope or inside temperature. The air change rate follows different probability distributions depending on the contribution and the quantity of the stack, wind, or mechanical forces. The most typical are normal, lognormal, Weibull, and Gamma distributions.

The distribution of dynamic thermal transmittance, U, depends on the direction of airflow through a structure and is specified separately for the cases of exfiltration and infiltration. It has been assumed that the average thermal transmittance U^0 is a normally distributed random variable dependent on the temperature and the density of insulation with mean value and standard deviation given by Equations 11. It means that the probabilistic description of variable U^0 already refers to its variations caused by the changes of the temperature of material.

Correlation coefficients between random variables ACH and U^0 are dependent on the case considered: infiltration or exfiltration. For the infiltration case, when the heating season is considered, the U^0 is negatively correlated with the airflow through the insulation, while, for the exfiltration case, the correlation is positive.

A functional relationship among ACH and U^0 and dynamic U-factor is established with the help of mathematical models. It is called the performance function (see Figure 1) and is described by Equations 15 and 16. They are obtained from Equations 3 to 6 and account for the following relationship between airflow rate Q (m^3/s) and air change rate (ACH) (1/h):

$$Q = \frac{ACH}{3600} V \tag{14}$$

where

V = ventilated volume of a building (m^3).

The performance function of a dynamic U-factor for the infiltration case is given by

$$U = -\frac{a \cdot ACH}{A} \frac{\exp\left(-\frac{a}{A}\frac{ACH}{U^0}\right)}{\exp\left(-\frac{a}{A}\frac{ACH}{U^0}\right) - 1} . \tag{15}$$

The performance function of a dynamic U-factor for the exfiltration case is given by

$$U = -\frac{a \cdot ACH}{A} \frac{\exp\left(-\frac{a}{A}\frac{ACH}{U^0}\right)}{\exp\left(\frac{a}{A}\frac{ACH}{U^0}\right) - 1} , \tag{16}$$

where

$$a = \frac{c_a \rho_a V}{3600} , \tag{17}$$

where

ρ_a = air density (kg/m^3),

c_a = specific heat capacity of air (at a constant pressure) (kJ/kgK), and

V = ventilated volume of a building (m^3).

Reliability Analysis of Dynamic U-Factor

The physical state of a building in the context of thermal performance is described by the U-factor. Undesired performance can appear as a response to the changes of physical state of the building caused by variations of load parameters: climatic, environmental (external load), structural parameters, building installations parameters, or occupant behavior (internal loads) (see Figure 1). Performance failure is associated with the U-factor being higher than a preselected performance criterion, also called *serviceability limit*. Limit state approach gives opportunity to calculate P_f, probability of the performance failure, or reliability, which is a complement to P_f.

The serviceability limit criterion for U-factor for a house can be obtained for example from the Swedish Building Code (Boverkets Byggregler 1994), where the maximum allowable value of overall thermal transmittance is stated as equal to

$$R_{U^0} = 0.18 + 0.95\frac{A_f}{A_{tot}}, \tag{18}$$

where

A_f = total area of windows and doors (m^2) and

A_{tot} = total area of a building envelope (m^2).

A serviceability requirement can be formulated as $U \le R_{U^0}$. A performance failure is then described by $U > R_{U^0}$.

Probability of performance failure given by Equation 19 is equal to the area under probability density function of thermal transmittance for $U > R_{U^0}$ (see Figure 5).

$$P_f = P(U > R_{U^0}) = 1 - P(U \le R_{U^0}) \tag{19}$$

In a similar way the serviceability limit can be considered for a dynamic U-factor. However, it is important to remember that the limits evaluated for the two cases (infiltration and exfiltration) can be different. They should be stated in relation to the effect of the heating system. Analyzing the probability of failure for different heating demands and for different constructions of a building exposed to the climatic conditions typical for the site, one can choose the reliable heating solution (with acceptable likelihood of performance failure). Reliability analysis can then become a tool helping in dimensioning of the heating system.

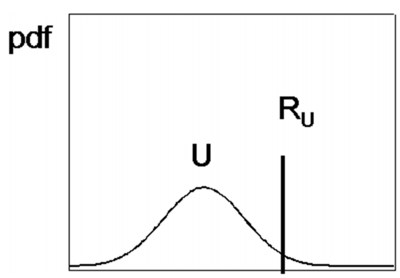


Figure 5 *Concept of the serviceability limit for thermal transmittance: pdf (probability density function).*

CASE STUDY

The object of our study is a timber-framed low-rise naturally ventilated building with aspect ratio 2 and slope of the roof of 45°. The building envelope is homogeneous. Two building sites situated in the districts of Luleå and Göteborg are considered. Examples are made for wind blowing from the west and from the south. The site is described as a semi-urban area with the surface roughness equal to 0.3 m. It is assumed that the building is surrounded by other obstructions (other buildings, topography, vegetation) equivalent to half of its height. The following input data are used:

- Building-related deterministic input data:
  1. Dimensions: volume of the house of 486 m^3, total area of the envelope of 336 m^2.
  2. Leakage characteristics of the house: The envelope from the south side is five times more leaky than from the west side.
  3. Thermal properties of the envelope are characterized by the area-weighted thermal transmittance (see Equation 12) equal to 0.359 W/m^2K.
- Climate-related random input data:
  One-hour mean wind speed and one-hour mean external temperature are the two random variables. Parameters of the distribution of each variable are estimated on the basis of a 40-year record of observations made on meteorological stations at the airports in Luleå and Göteborg. Wind speed data were transformed to the house location. External temperature at the building site is assumed to be equal to the temperature measured at the meteorological station. Probabilistic characteristics of local wind speed and external temperature are given in Table 1.
  A constant internal temperature in the building of 20°C is assumed. The difference between internal and external temperature at the site is given in the form of parameters of truncated normal distribution evaluated for the "heating" periods (when $T_{ext} < T_{int}$), when wind was blowing from the specified direction sectors. The correlation

Table 1. Statistical Parameters of the Temperature Difference $\Delta T = T_{int} - T_{ext}$, the Wind Speed at the Site v_d, and the Correlation Coefficient between the Wind Speed and the Temperature Difference $\rho_{\Delta T,v}$ for the Periods When Wind Was Blowing from the South (S) and from the West (W)

Building Site	d	Mean v_d	Standard Deviation v_d	Mean ΔT	Standard Deviation ΔT	$\rho_{\Delta T,v}$
Göteborg	W	3.73	2.18	8.90	6.13	0
Göteborg	S	3.51	2.13	12.13	6.55	0
Luleå	W	2.15	1.51	17.09	9.60	0.37
Luleå	S	2.37	1.44	15.30	9.46	0.18

between the temperature difference and wind speed is assumed to be equal to the correlation between external temperature and wind speed and is given in Table 1.

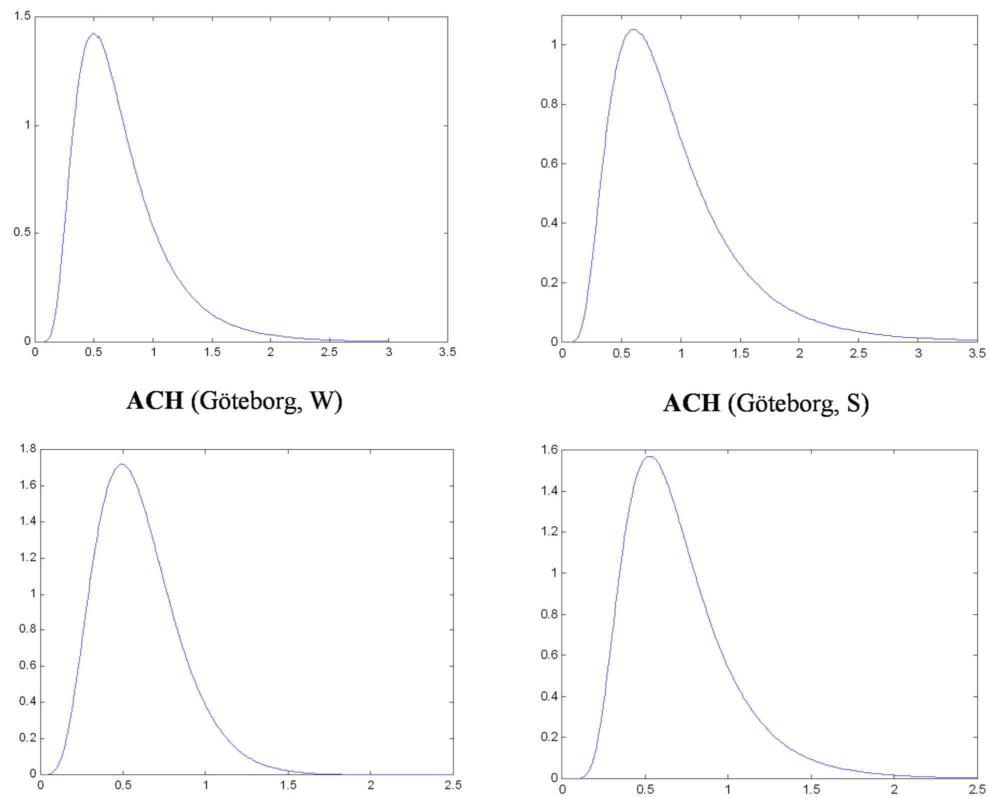
The results of estimation of air change rate distributions for the combined effect of temperature and wind-driven air exchange for wind blowing from the south and from the west are presented for two building sites, Luleå and Göteborg, in Table 2 and Figure 6. The mean value and the standard deviation of the air change rate, as well as a type of distribution recommended according to the Kolmogorov-Smirnov test, are given (the first and the second choices are listed).

The distribution of a total air change rate depends on the contribution of the stack and wind to the exchange of air in the building. An increased contribution of wind in the process of exchanging the air can be caused by specific climatic conditions or by special leakage characteristics of the facades of a building exposed to wind, as it is the case for the test house analyzed.

For a dominant influence of a stack effect, as is observed for a western wind for a building site in Luleå (see Tables 1 and 2), a total air change rate can be described by a Gamma or a log-normal distribution. For a dominant influence of wind, represented by a building site in Göteborg, the air change rate is well described by a log-normal distribution for both the south and the west wind directions. Where the ACH caused by

Table 2. Probabilistic Description of Random Variables: U^0 and ACH as the Input Data to the FORM Approximations of U-Factor

Building Site	Wind Direction	Mean U^0	Standard Deviation U^0	Mean ACH	Standard Deviation ACH	Type of Distribution of ACH
Göteborg	W	0.353	0.004	0.728	0.382	Log-normal (Gamma)
Göteborg	S	0.351	0.004	0.945	0.546	Log-normal (Gamma)
Luleå	W	0.348	0.006	0.598	0.252	Gamma (log-normal)
Luleå	S	0.349	0.006	0.706	0.327	Log-normal (Gamma)



ACH (Göteborg, W)

ACH (Göteborg, S)

ACH (Luleå, W)

ACH (Luleå, S)

Figure 6 *Probability density functions of the air change rate estimated for the Göteborg and Luleå sites for periods when wind was blowing from the west and from the south using the PROMO model.*

wind and the ACH caused by stack effect are comparable (see the case Luleå, wind from the south), a log-normal distribution of ACH is also fitted.

The thermal transmittance of the tight envelope U^0 is treated in the model as a normally distributed random variable with the mean value and the standard deviation described by Equation 11 and given in Table 2. For a leaky envelope enabling filtration of the air through the insulation, thermal transmittance varies with the changes of the airflow (see Equations 3 to 5). Changes of the thermal transmittance characterized by dynamic U-factor are described for the case of infiltration and exfiltration by probability density functions of the dynamic U-factor. These functions have been approximated (using FORM) for the test house situated in Luleå and Göteborg. The input data are given in Table 2. Correlation coefficient between ACH and U^0 is assumed to be equal to –0.9 for the infiltration case and 0.9 for the exfiltration case.

The results of probabilistic approximations of dynamic U-factor are presented in Table 3. The probability density functions of dynamic U-factors are shown in Figure 7 for the case of infiltration and in Figure 8 for the case of exfiltration. The mean μ and the standard deviation σ of the U-factor are given in each picture.

Table 3. The Results of a Simulation of a Distribution of Dynamic U-Factor

Case	Building Site	Wind Direction	Mean U	Standard Deviation U	Skewness	Type of Distribution of ACH
Infiltration	Göteborg	W	0.21	0.060	-0.57	Weibull (normal)
	Göteborg	S	0.18	0.068	-0.35	normal (Weibull)
	Luleå	W	0.23	0.046	-0.57	Weibull (normal)
	Luleå	S	0.21	0.054	-0.51	Weibull (normal)
Exfiltration	Göteborg	W	0.57	0.140	1.81	3-p Gamma (Gumbel)
	Göteborg	S	0.64	0.196	1.79	3-p Gamma (Gumbel)
	Luleå	W	0.52	0.087	1.62	Gumbel (3-p Gamma)
	Luleå	S	0.55	0.118	1.82	Gumbel (3-p Gamma)

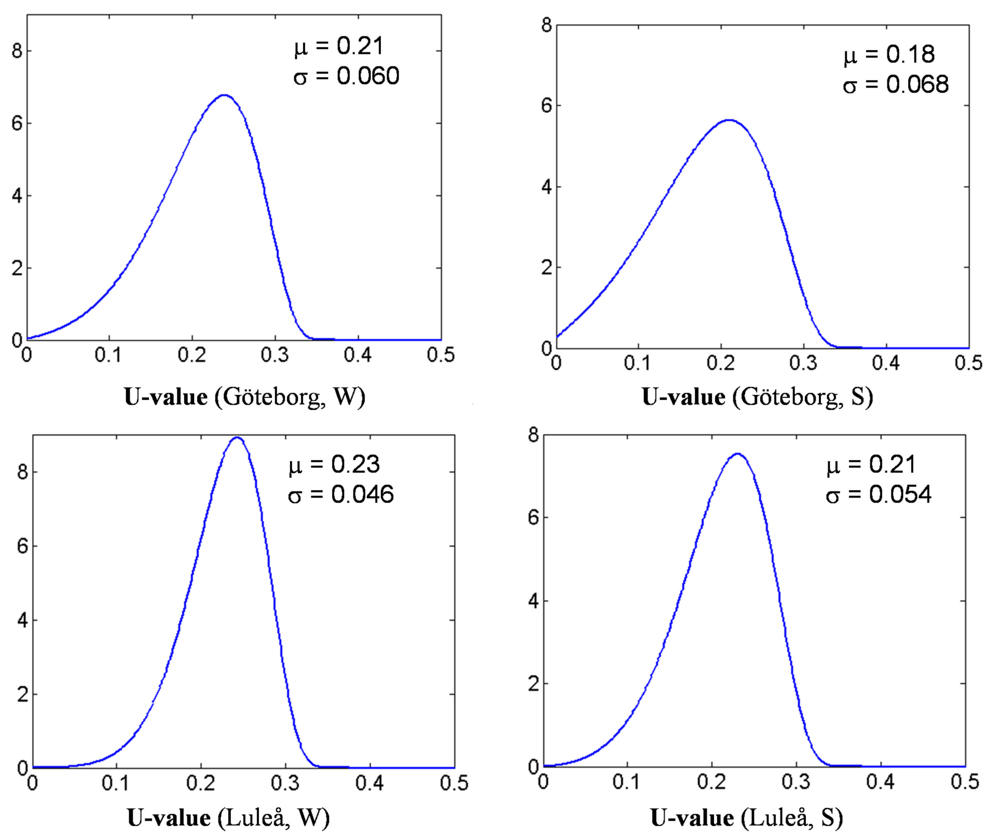


Figure 7 *Probability density functions of dynamic U-factor for the case of infiltration approximated for the building located near Göteborg (up) or Luleå (down) for western winds (left) and for southern winds (right).*

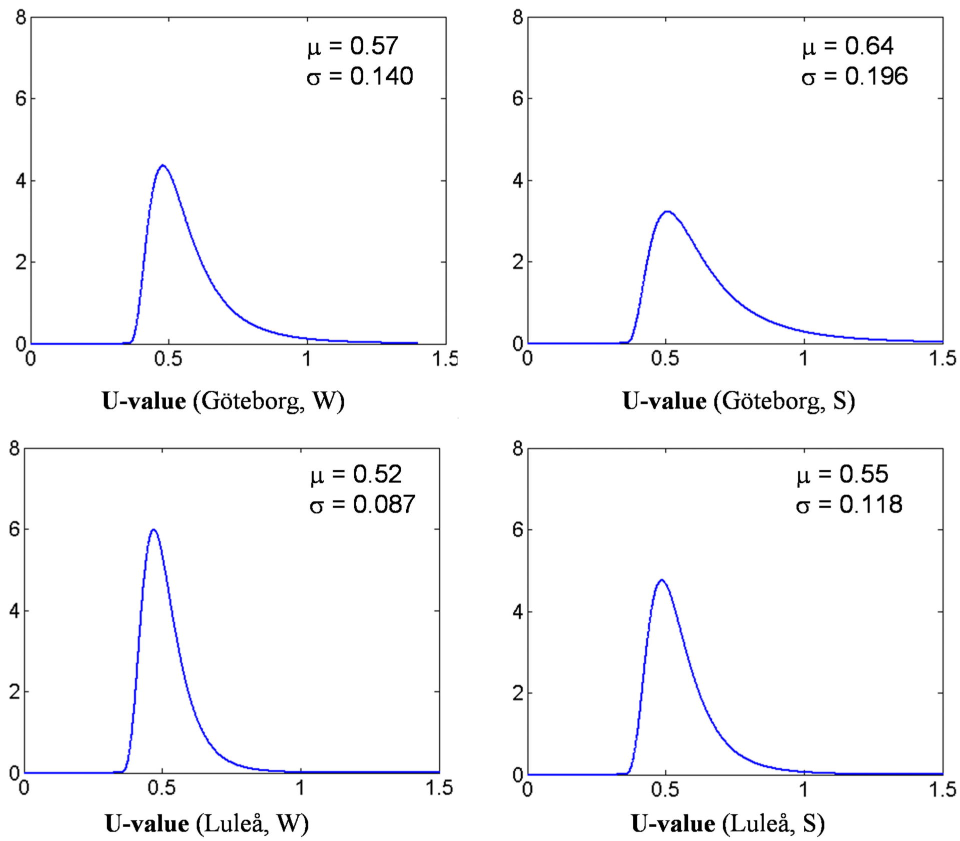


Figure 8 *Probability density functions of dynamic U-factor for the case of exfiltration approximated for the building located near to Göteborg (up) or Luleå (down) for western winds (left) and southern winds (right).*

For the case of Göteborg, southern wind, the air change rate is higher than for other cases (see Table 2), and the influence on the mean value and the standard deviation of the thermal transmittance is more significant (see Table 3). This is due to a combined effect of higher wind velocity and more leaky envelope of the southern face of the building.

Generally, one can notice that the probability density functions of the dynamic U-factor approximated for the infiltration are skewed to the left, while these approximated for the exfiltration are skewed to the right (see Table 3). This is because of the specific character of the relationship between Nusselt number and air change rate (see Equations 3 to 5). It also influences the family of distributions, giving the best fit to dynamic thermal transmittance. For the case of infiltration, the best fits according to the Kolmogorov-Smirnov test have been obtained for the Weibull family of distributions while, for the exfiltration case, the three-parameter Gamma or, alternatively, Gumbel distributions have been fitted with the best results (see Table 3).

SUMMARY AND DISCUSSION

A probabilistic model of dynamic U-factor has been presented. It takes into account only some of the uncertainties related to the climatic load of a building site and the internal load coming from the building installations and occupant behavior. The model can be used to estimate a probability distribution of the effective (actual) U-factor of the building envelope accounting for the properties of the thermal insulation and stochastic characteristics of the microclimatic load. The model should be further developed to include uncertainties caused by other mechanisms and factors such as, for example, influence of wind or radiation on external heat transfer coefficient or the influence of nonhomogenity of the material characteristics (for example, density or thickness of insulation).

Probability density functions of thermal transmittance depend on the direction of the airflow through the envelope as well as on the shape of the air change rate distribution. The model uses probability density functions of air change rate approximated with the help of FORM techniques and the probabilistic model of air infiltration developed by Pietrzyk (2000). Depending on the relative contribution of the natural forces (wind, temperature) and mechanical forces the air change rate follows different probability distributions (normal, lognormal, Weibull, and Gamma). Generally, the probability density functions of the dynamic U-factor approximated for the infiltration are skewed to the left, while these approximated for the exfiltration are skewed to the right. This is because of the specific character of the relationship between Nusselt number and air change rate. It also influences the family of distributions, giving the best fit to dynamic thermal transmittance. For the case of infiltration, the best fits according to the Kolmogorov-Smirnov test have been obtained for the Weibull family of distributions, while, for the exfiltration case, the three-parameter Gamma (or, alternatively, Gumbel) distributions have been fitted with the best results (see Table 3).

The probabilistic approach can be applied even at the design stage to estimate thermal performance of a building envelope. The actual heating load for a building with lightweight construction with insulation, exposed to the climatic conditions typical for the site, can be predicted accounting for the variations of the thermal transmittance of the building components. Reliability analysis of U-factor can become a tool helping in dimensioning of the heating system.

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